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**Monetary Policy in a Channel System**

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# Monetary Policy in a Channel System<sup>\*</sup>

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## Abstract

This paper studies the theoretical properties of a channel system of interest-rate control in a dynamic general equilibrium model. Agents are subject to liquidity shocks which can be partially insured in a secured money market, or at a standing facility operated by the central bank. We show that it is optimal to have a strictly positive interest rate corridor and that a shift of the corridor affects the money market rate one for one. Moreover, the central bank can tighten its policy without changing its policy rate by simply increasing the corridor symmetrically around the policy rate.

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# 1 Introduction

Channel systems are becoming increasingly popular. Several central banks now implement monetary policy using a channel system and others are using at least some features of the channel system.<sup>1</sup> Despite its popularity, the consequences of implementing monetary policy with a channel system are not well understood. How does implementation of monetary policy in a channel system differ from plain-vanilla open market operations? What is the welfare maximizing policy? The purpose of this paper is to study the theoretical properties of a channel system.

In a channel system a central bank offers two standing facilities: a lending facility where it is ready to supply money overnight at a given lending rate against collateral and a deposit facility where banks can make overnight deposits to earn a deposit rate. The interest-rate corridor - defined by the difference between the lending and the deposit rates - is chosen to keep the overnight interest rate in the money market close to the target rate. In a pure channel system a change in policy is implemented by simply shifting the corridor without any open market operations.

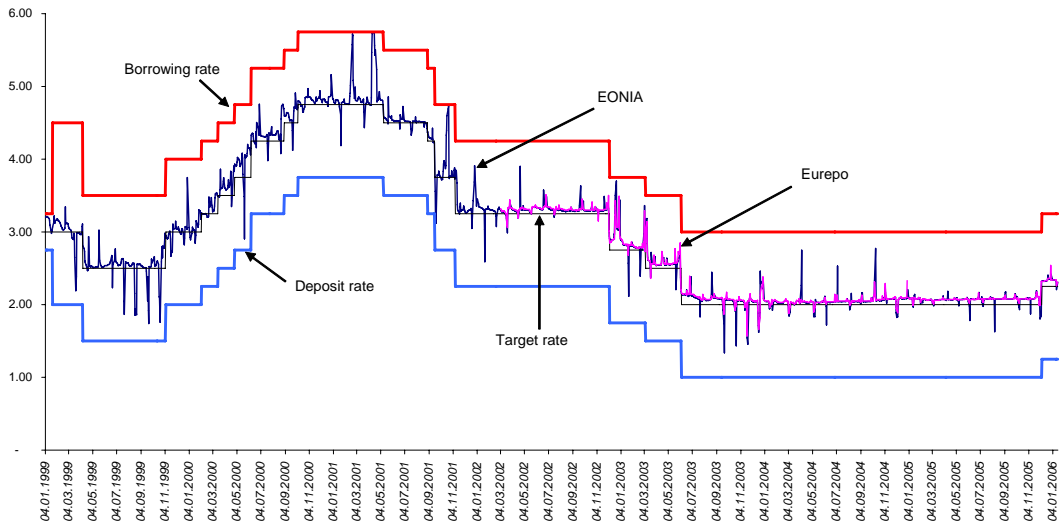
There are several stylized facts of channel systems that a reasonable theoretical model has to explain. First, all central banks set a strictly positive corridor. Second, central banks typically react to changing economic conditions by shifting the interest-rate corridor. Third, the money market rate tends to be in the middle of the corridor. We construct a general equilibrium model that is able to explain these stylized facts.

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<sup>1</sup>For example, versions of a channel system are operated by the Bank of Canada, the European Central Bank, the Reserve Bank of Australia, or the Reserve Bank of New Zealand. The US Federal Reserve System recently modified the operating procedures of its discount window facility in a way that it now shares elements of a standing facility. Prior to 2003, the discount window rate was set below the target federal fund rate, but banks faced penalties when accessing the discount window. In 2003 the Federal Reserve decided to set the discount window rate 100 basis points above the target federal fund rate and eased access conditions to the discount window. The resulting framework is similar to a channel system, where the deposit rate is zero and the lending rate 100 basis point above the target rate.

Moreover, we shed some light on the following questions. First, why do central banks choose different corridors? Most central banks choose a corridor of 50 basis points (e.g. Australia, Canada and New Zealand), while the European Central Bank's (ECB) lending rate is 200 basis points higher than its deposit rate (Figure 1).<sup>2</sup> Second, why can some central banks control the overnight interest rate very tightly while others cannot? For instance, the overnight interbank cash rate in New Zealand is almost always on the policy rate set by the Reserve Bank (Figure 2). In contrast, the Euro overnight rate fluctuates considerably around the target rate set by the ECB.

**Figure 1: EONIA - Euro OverNight Index Average  
and Eurepo - reference rate for the Euro GC repo market**  
*Source: European Banking Federation and ECB*

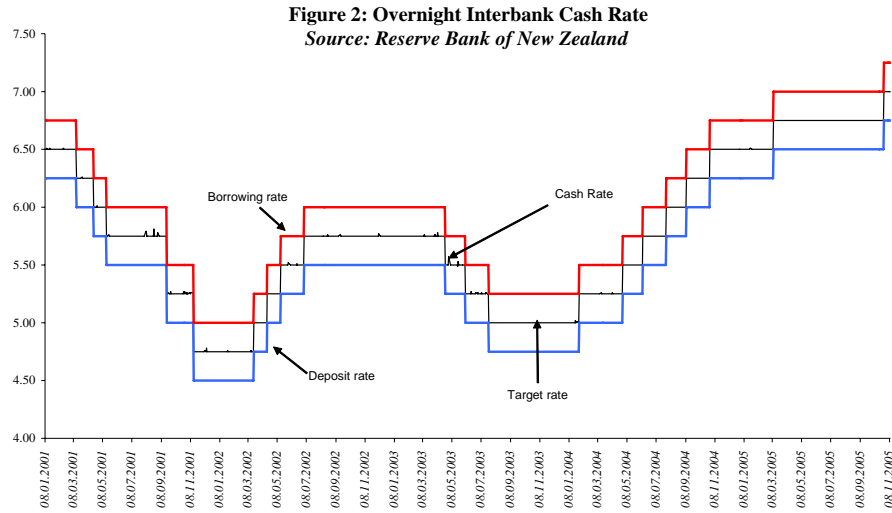


To study the previous observations we construct a dynamic general equilibrium model of a channel system with a money market and a welfare optimizing central bank. Agents are subject to idiosyncratic trading shocks which generate random liquidity needs.<sup>3</sup> These shocks can be partially insured in a secured money market.

<sup>2</sup>As can be seen from Figure 1, the ECB increased its spread dramatically from 50 basis points to 250 basis points around February 1999 before reducing it to 200 basis points around April 1999.

<sup>3</sup>We abstract from modelling commercial banks explicitly. Rather, we assume that households

To provide further insurance the central bank operates a standing facility where agents can borrow or deposit money at the specified rates. In accordance with central bank practice, there is no limit to the size of deposits on which interest is paid. There is also no limit to the size of a loan that an agent can obtain provided that the loan is fully collateralized. Within this framework we answer the following three questions. First, what is the welfare maximizing interest-rate corridor? Second, what is the optimal collateral policy? Third, how does changing the corridor affect the money market rate?



The following results emerge from our model. We show that it is optimal to have a positive wedge between the borrowing rate and the deposit rate if the opportunity cost of holding collateral is positive.<sup>4</sup> The optimal corridor is decreasing in the rate of return of the collateral and equal to zero when the opportunity cost of acquiring have direct access to the money market and the central bank's lending and deposit facility. The trading shocks are an approximation for liquidity shocks faced by commercial banks after trading in the money market. Since there is no trading of reserves feasible after this market has closed, banks who need liquidity have no choice but to use the standing facility offered by the central bank.

<sup>4</sup>The rate of return of the collateral determines the opportunity costs for commercial banks of accessing the lending facility of the central bank where a high rate of return implies a small or zero

collateral is zero. We also show that a central bank has two equivalent options to implement a given policy: it can either shift the corridor while keeping the corridor constant, or it can change the corridor. For instance, it can keep the deposit rate constant and increase the borrowing rate. In particular, it can set a zero deposit rate and only modify the lending rate as it is done for example by the US Federal Reserve System.

An interesting aspect of the channel system is that a central bank can tighten its policy without changing its target rate. The reason is that by increasing the corridor symmetrically around the target rate the central bank worsens the option for banks of accessing the standing facility. As a result the policy regime is tighter. This result indicates that the ECB with its 200 basis points corridor implements a tighter monetary policy than the other central banks operating a channel system mentioned before. This property of the channel system might explain why countries with the same target rate but different corridors perform differently.

We also find that the money market rate tends to be above the target rate if the opportunity cost of holding collateral are positive or/and if there is too little liquidity. This property of the model provides an answer to the ECB's concern about the fact that the Euro overnight rate tends to be above the target rate (Figure 1). Our model thus suggests that in the channel system operated by the ECB either holding collateral is too costly or liquidity is scarce.

**Literature** There are very few theoretical studies related to our paper and all of them are partial equilibrium models. An early contribution is the model of banks reserve management under uncertainty by Poole (1968). Woodford (2000, 2001) discusses and analyses the channel system to address the question of how to conduct monetary policy in a world with a vanishing stock of money. Whitsell (2006) evaluates reserves regimes versus channel systems. Elements of channel systems have been

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opportunity cost. Assets accepted as collateral are typically low-risk and low-yield assets such as government securities.

previously described in Gaspar, Quiros and Mendizabal (2004), Guthrie and Wright (2000), and Heller and Lengwiler (2003). These studies are complementary to our approach. We consider a general equilibrium model where a positive liquidity shock for one bank corresponds to a negative liquidity shock for another bank. The cost of pledging collateral is explicit and money is essential.<sup>5</sup> Finally, we conduct a welfare analysis and derive the welfare maximizing interest-rate corridor.

The main reason why there is no other general equilibrium analysis is that money growth is endogenous in a channel system. In contrast, most theoretical models of monetary policy characterize optimal policy in terms of a path for the money supply. In practice, however, monetary policy involves rules for setting nominal interest rates and most central banks specify operating targets for overnight interest rates. This paper therefore is an attempt to break the apparent dichotomy (Goodhard, 1989) between theoretical analysis and central bank practices.

The paper is structured as follows. Section 2 outlines the environment. The equilibrium without money market and the optimal monetary policy is characterized in Section 3. The equilibrium with the money market is characterized in Section 4. Section 5 concludes. All proofs and a description of the Euro money markets and the ECB's operating procedures can be found in the Appendix.

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<sup>5</sup>By essential we mean that the use of money expands the set of allocations (Kocherlakota 1998 and Wallace 2001).

## 2 Environment

We construct a dynamic general equilibrium model with a  $[0,1]$ -continuum of infinitively-lived agents and a central bank. Time is discrete and in each period three perfectly competitive markets open sequentially.<sup>6</sup> The first market is the settlement market where all agents produce and consume a general good and settle their claims from the previous period. The second market is the money market where agents can borrow and lend cash at the market rate and in the third market agents either produce or consume a perishable good.

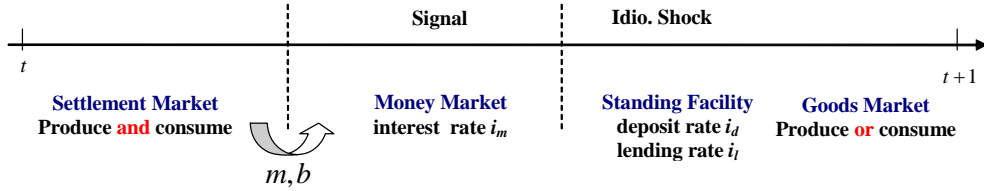


Figure 3: Sequence of markets.

General goods in the first market are produced solely from inputs of labor according to a constant return to scale production technology where one unit of the consumption good is produced with one unit of labor generating one unit of disutility. Thus, producing  $h$  units of the general good implies disutility  $-h$ , while consuming  $h$  units gives utility  $h$ .<sup>7</sup>

At the beginning of the third market, agents receive idiosyncratic preference and technology shocks which determine whether they consume or produce in this market. With probability  $1 - n$  an agent can consume and cannot produce. We refer to these agents as buyers. With probability  $n$ , an agent can produce and cannot consume. These are sellers. Agents get utility  $u(q)$  from  $q$  consumption in the second market,

<sup>6</sup>The sequence of markets is motivated by the ECB's operating procedures. In the Appendix we describe the functioning of the Euro money markets and the ECB's operating procedures.

<sup>7</sup>The environment shares elements of Berentsen, Camera and Waller (2005) (credit market), Koepl, Monnet and Temzelides (2006) (settlement) and Lagos and Wright (2005) (degenerate distribution of money holdings).



where  $u'(q) > 0$ ,  $u''(q) < 0$ ,  $u'(0) = +\infty$  and  $u'(\infty) = 0$ . Producers incur a utility cost  $c(q) = q$  from producing  $q$  units of output. All trades are anonymous and agents' trading histories are private information. Since sellers require immediate compensation for their production effort money is essential for trade. The discount factor is  $\beta$  where for technical reasons we assume that  $\beta > n$ .

**Money market** At the beginning of the money market, agents receive a signal about the probability that they will become a consumer or a producer in the third market. With probability  $\sigma^k$  an agent receives the information that he will be a seller with probability  $n^k$ ,  $k = H, L$ , where  $\varepsilon \equiv n^H - n^L \in [0, 1]$ . We assume that  $n = \sum_{k=H,L} \sigma^k n^k$  so that there is no aggregate uncertainty. This modelling approach captures the idea that when the money market is open agents receive information about their end of day cash holdings. Some agents believe that they are likely to have excess cash at the end of the day and others that they are likely to be short of cash. The difference in expected liquidity needs generate an incentive for trading in the money market.

There are three cases. When  $\varepsilon = 0$  the signal contains no information and so agents have no gains from trading in the money market. Consequently, no trade occurs in the money market. We will consider this case in Section 3. If  $\varepsilon = 1$  there is no uncertainty about the liquidity shock in the goods market. Consequently, the portfolios are completely adjusted in the money market and no agent accesses the standing facility. Finally, if  $\varepsilon \in (0, 1)$ , the signal contains some information about the future liquidity shock, but the information is not perfect. As a result agents use both the money market and the standing facility to adjust their portfolio. For example, some agents will get the information that they will be sellers with high probability but then turn out to be buyers. These agents will first use the money market to trade away their cash and then use the standing facility to take out loans. We will consider this case in Section 4.

**Standing facility** We assume that at the beginning of the third market after the idiosyncratic shocks are observed a central bank offers a borrowing facility and a deposit facility. It offers nominal loans  $\ell$  at an interest rate  $i_\ell$  and promises to pay interest rate  $i_d$  on nominal deposits  $d$  with  $i_\ell \geq i_d$ . This condition eliminates the possibility for arbitrage where agents borrow and subsequently make a deposit at interest  $i_d > i_\ell$ , thus increasing their money holdings at no cost. The central bank operates at zero cost.

Since we focus on standing facilities, we restrict financial contracts to overnight contracts. An agent who borrows  $\ell$  units of money from the central bank in market 2, repays  $(1 + i_\ell)\ell$  units of money in market 1 of the following period. Also, an agent who deposits  $d$  units of money at the central bank in market 2 of period  $t$  receives  $(1 + i_d)d$  units of money in market 1 of the following period.

Accordingly, the money stock evolves endogenously as follows

$$M_{+1} = M - (1 - n)i_\ell\ell + ni_d d + \pi M, \quad (1)$$

where  $M$  denotes the per capita stock of money at the beginning of period  $t$ . In the first market total loans  $(1 - n)\ell$  are repaid. Since interest rate payments by the agents are  $(1 - n)i_\ell\ell$ , the stock of money shrinks by this amount. Interest payments by the central bank on total deposits are  $ni_d d$ . The central bank simply prints additional money to make these interest payments so the stock of money increases by this amount. Finally, the central bank can also change the stock of money via lump-sum transfers  $T = \pi M$  in market 1. However, since central banks cannot tax agents, we restrict these lump sum transfers to be positive, that is  $\pi \geq 0$ .<sup>8</sup>

**Default** In any model of credit, default is a serious issue. Since production is costly, those agents who have borrowed in the previous period have an incentive to default

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<sup>8</sup>The lump-sum transfers are a substitute for open-market operations that we do not model here. However, in pure channel systems central banks do not use open-market operations to affect the money market rate on a regular basis. Nevertheless, there is no clear reason why we should rule this possibility out. Later we will show that it is optimal to set  $\pi = 0$ .

in market 1 of the current period. To prevent default all loans must be secured with collateral. We assume that the central bank operates the money market and keeps track of all financial arrangements and collateral holdings. In particular, only the central bank can verify the existence of collateral. This means that collateral cannot be used to secure IOU's in the goods market.

We assume that general goods produced in market 1 can be stored with a constant return to scale technology that yields  $R \geq 1$  units of general goods in market 1 of the following period. We also impose  $\beta R \leq 1$  since when  $\beta R > 1$  agents would store infinite amounts of goods which is inconsistent with equilibrium.

**First-best allocation** In the Appendix we show that the expected lifetime utility of the representative agent for a stationary allocation  $(q, b)$  where  $q$  is consumption and  $b$  collateral holdings at the beginning of a period is given by

$$(1 - \beta) \mathcal{W} = (1 - n) [u(q) - q] + (\beta R - 1) b \quad (2)$$

The first term on the right-hand side is the expected utility from consuming and producing the market 3 good. The second term is the utility of producing collateral and receiving the return in the following period.

It is obvious that the first-best allocation  $(q^*, b^*)$  satisfies  $q = q^*$  where  $q^*$  is the value of  $q$  that solves  $u'(q) = 1$ . Moreover,  $b^* = 0$  if  $\beta R < 1$  and  $b^*$  is indeterminate if  $\beta R = 1$ . Thus, a social planner would never choose a positive amount of collateral when collateral is costly.

### 3 No trade in the money market

Assume  $\varepsilon = 0$ . Then there is no trade in the money market and agents only use the lending and deposit facilities of the central bank to adjust their money holdings. We now characterize the symmetric stationary equilibrium in this case.

In period  $t$ , let  $\phi \equiv 1/P$  be the real price of money in market 1. We focus on symmetric and stationary equilibria where all agents follow identical strategies and where the real allocation is constant over time. In a stationary equilibrium beginning-of-period real money balances are time invariant

$$\phi M = \phi_{+1} M_{+1}. \quad (3)$$

This implies that  $\phi/\phi_{+1} = P_{+1}/P = M_{+1}/M = \gamma$ . Moreover, we restrict our attention to stationary equilibria where  $\gamma$  is time invariant which eliminates stationary equilibria where  $\gamma$  is stochastic.

We let  $V(m, b)$  denote the expected value from entering market 2 with  $m$  units of money and  $b$  collateral.  $W(m, b, \ell, d)$  denotes the expected value of entering the first market with  $m$  units of money,  $b$  collateral,  $\ell$  loans, and  $d$  deposits. For notational simplicity we suppress the dependence of the value function on the time index  $t$ .

In what follows we look at a representative period  $t$ .

### 3.1 Settlement

In the first market, the problem of a representative agent is:

$$\begin{aligned} W(m, b, \ell, d) &= \max_{h, m_2, b_2} -h + V(m_2, b_2) \\ s.t. \quad \phi m_2 + b_2 &= h + \phi m + Rb + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell + \phi\pi M. \end{aligned}$$

where  $h$  is hours worked in market 1. Using the budget constraint to eliminate  $h$  in the objective function, one obtains the first-order conditions<sup>9</sup>

$$V_m = \phi \quad (4)$$

$$V_b \leq 1 \quad (= \text{ if } b > 0) \quad (5)$$

$V_m \equiv \frac{\partial V(m_2, b_2)}{\partial m_2}$  is the marginal value of taking an additional unit of money into the second market in period  $t$ . Since the marginal disutility of working is one,  $-\phi$  is the

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<sup>9</sup>We focus on monetary equilibria where (4) holds with equality. In contrast, there are monetary equilibria where agents do not use the standing facility implying  $b = 0$  because  $V_b < 1$ .

utility cost of acquiring one unit of money in the first market of period  $t$ .  $V_b \equiv \frac{\partial V(m_2, b_2)}{\partial b_2}$  is the marginal value of taking additional collateral into the second market in period  $t$ . Since the marginal disutility of working is 1,  $-1$  is the utility cost of acquiring one unit of collateral in the first market of period  $t$ . The implication of (4) and (5) is that all agents enter the following period with the same amount of money and the same quantity of collateral (which can be zero). This is the reason why we interpret this market as a settlement stage. By itself, this market does not increase social welfare. Rather, it involves a mere transfer of an asset between participants in order to settle claims from the previous period.

The envelope conditions are

$$W_m = \phi; W_b = R; W_\ell = -\phi(1 + i_\ell); W_d = \phi(1 + i_d) \quad (6)$$

where  $W_j$  is the partial derivative of  $W(m, b, \ell, d)$  with respect to  $j = m, b, \ell, d$ .

### 3.2 Liquidity shocks

We immediately proceed to market 3 since when  $\varepsilon = 0$  no trade occurs in the money market. At the beginning of market 3, agents receive idiosyncratic shocks which determine whether they are consumers or producers. With probability  $1 - n$  an agent becomes a consumer and with probability  $n$  a producer. Let  $q$  and  $q_s$  respectively denote the quantities consumed by a buyer and produced by a seller in market 3. Let  $\ell_b$  ( $\ell_s$ ) and  $d_b$  ( $d_s$ ) respectively denote the loan obtained and the amount of money deposited by a buyer (seller) in market 2. An agent who has  $m$  money and  $b$  collateral at the opening of market 3 has expected lifetime utility

$$\begin{aligned} V(m, b) = & (1 - n)[u(q) + \beta W(m - pq - d_b + \ell_b, b, \ell_b, d_b)] \\ & + n[-q_s + \beta W(m + pq_s - d_s + \ell_s, b, \ell_s, d_s)] \end{aligned}$$

where  $q, q_s, \ell_s, \ell_b, d_s$  and  $d_b$  are chosen optimally as follows.

It is obvious that buyers will never deposit funds in the central bank and sellers will never take out loans and therefore  $d_b = 0$  and  $\ell_s = 0$ . To simplify notation let

$\ell = \ell_b$  and  $d = d_s$ . Accordingly, we get

$$\begin{aligned} V(m, b) = & (1 - n)[u(q) + \beta W(m - pq + \ell, b, \ell, 0)] \\ & + n[-q_s + \beta W(m + pq_s - d, b, 0, d)] \end{aligned}$$

where  $q_s$ ,  $q$ ,  $\ell$  and  $d$  solve the following optimization problems.

A seller's problem is  $\max_{q_s, d} [-q_s + \beta W(m + pq_s - d, b, 0, d)]$  s.t.  $m + pq_s - d \geq 0$ .<sup>10</sup>

Using (6), the first-order condition reduces to

$$p\beta\phi_{+1} + p\beta\phi_{+1}\lambda_d = 1 \quad (7)$$

$$i_d = \lambda_d \quad (8)$$

where  $\beta\phi_{+1}\lambda_d$  is the multiplier on the deposit constraint. The two conditions can be combined to get

$$p\beta\phi_{+1}(1 + i_d) = 1. \quad (9)$$

If an agent is a buyer, he solves the following maximization problem:

$$\begin{aligned} \max_{q, \ell} \quad & u(q) + \beta W(m - pq + \ell, b, \ell, 0) \\ \text{s.t.} \quad & pq \leq m + \ell \text{ and } \ell \leq \bar{\ell} \end{aligned}$$

where

$$\bar{\ell} = Rb / [\phi_{+1}(1 + i_\ell)] \quad (10)$$

is the maximal amount that a buyer can borrow from the central bank since  $b$  units of collateral transform into  $Rb$  units of real goods at the beginning of the following period. These goods can be sold for  $Rb/\phi_{+1}$  units of money. Finally, the collateral must also cover the interest payment.

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<sup>10</sup>Here we assume that sellers can deposit their money holdings at the standing facility, including the proceeds from their latest transaction. This is in line with the institutional details described in the Appendix that banks can access the standing facility of the ECB 30 minutes after the close of the money market. The results are not fundamentally affected when agents can only deposit a fraction or none of their receipts from selling goods.

Using (6) the buyer's first-order conditions can be written as

$$u'(q) = p\beta\phi_{+1}(1 + \lambda_q) \quad (11)$$

$$\lambda_q = \lambda_\ell + i_\ell \quad (12)$$

where  $\beta\phi_{+1}\lambda_q$  is the multiplier of the buyer's budget constraint and  $\beta\phi_{+1}\lambda_\ell$  the one of the borrowing constraint. Using (9) and combining (11) and (12) yields

$$u'(q) = \frac{1 + i_\ell + \lambda_\ell}{1 + i_d}. \quad (13)$$

If the borrowing constraint is not binding and the central bank sets  $i_\ell = i_d$ , trades are efficient. If the borrowing constraint is binding, then  $u'(q) > 1$  which means trades are inefficient even when  $i_\ell = i_d$ .

Using the envelope theorem and (11), the marginal value of money in market 3 is

$$V_m = (1 - n)u'(q)/p + n\beta\phi_{+1}(1 + i_d). \quad (14)$$

The marginal value of money has a straightforward interpretation. An agent with an additional unit of money becomes a buyer with probability  $1 - n$  in which case he acquires  $1/p$  units of goods yielding additional utility  $u'(q)/p$ . With probability  $n$  he becomes a seller in which case he deposits overnight his money yielding the nominal return  $1 + i_d$ . Note that the standing facility increases the marginal value of money because agents can earn interest on idle cash.

### 3.3 Liquidity premium

Since in equilibrium there is no default the real return of collateral is  $\beta R$ . The real return is smaller than the marginal value  $V_b$  if  $\lambda_\ell > 0$ . To see this, use the envelope theorem to derive the marginal value of collateral in the second market

$$V_b = (1 - n)\lambda_\ell\beta R / (1 + i_\ell) + \beta R. \quad (15)$$

Thus, the difference between the real return and the marginal value is  $(1 - n)\lambda_\ell\beta R / (1 + i_\ell)$  which is positive if collateral relaxes the borrowing constraints of the buyers. It is

critical for the working of the model that  $V_b > \beta R$ . The reason is that, since  $\beta R - 1$  is negative, agents are only willing to hold collateral if the liquidity value as expressed by the shadow price  $\lambda_\ell$  is positive.

To derive the liquidity premium on the collateral use the first-order conditions (5) and (13) to write (15) as follows:

$$1 - \beta R = (1 - n) [u'(q)\beta R/\Delta - \beta R]. \quad (16)$$

where  $\Delta \equiv (1+i_\ell)/(1+i_d)$ . The term  $\beta R/\Delta$  is the price of goods in terms of collateral in market 3. A buyer can use the collateral to borrow  $\frac{R}{\phi_{+1}(1+i_\ell)}$  units of money which allows him to acquire  $\frac{R}{p\phi_{+1}(1+i_\ell)} = \frac{\beta R(1+i_d)}{1+i_\ell} = \beta R/\Delta$  units of goods.

The right-hand side of equation (16) is the liquidity premium on the collateral. While collateral costs  $-1$  to produce, its return is  $\beta R \leq 1$ . Hence, if  $\beta R < 1$ , agents need an incentive to hold collateral. This is provided by making collateral liquid.

If the return on the collateral increases, then, holding  $q$  constant, its liquidity premium will increase. To satisfy (16) the marginal benefit from an additional unit of collateral  $u'(q)/\Delta$  must fall which means that  $q$  must increase. In contrast, an increase in  $\Delta$ , holding  $q$  constant, reduces the liquidity premium since an increase in  $\Delta$  increases the cost of acquiring money with collateral. Consequently, to satisfy (16) the marginal benefit of an additional unit of good must rise and therefore  $q$  decreases. Monetary policy affects the allocation and welfare by its choice of  $\Delta$ .

### 3.4 Symmetric stationary equilibrium

To define a symmetric stationary equilibrium use the first-order condition (5) and (16) to get

$$\frac{1 - \beta R}{\beta R} \geq (1 - n) [u'(q)/\Delta - 1] \quad (= \text{ if } b > 0). \quad (17)$$

Then (4), (9), (14), and taking into account that in a stationary equilibrium  $M_{+1}/M = \phi/\phi_{+1} = \gamma$ , yield

$$\frac{\gamma - \beta(1+i_d)}{\beta(1+i_d)} = (1 - n) [u'(q) - 1]. \quad (18)$$



Also from (1) we get

$$\gamma = 1 + i_d - (1 - n)(i_\ell - i_d) \frac{z_\ell}{z_m} + \pi, \quad (19)$$

where  $z_m = m/p$  and  $z_\ell = \ell/p$ . To derive this equation we use  $d = m + pq_s$ , market clearing  $nq_s = (1 - n)q$  and we take into account that in symmetric equilibrium all agents hold identical amounts of money when they enter market 3. Then, from the budget constraint of the buyer we have

$$q = z_m + z_\ell. \quad (20)$$

Finally, since  $\beta R < 1$  in any equilibrium where agents hold collateral it must be the case that the borrowing constraint is binding and so from (9) and (10) we get<sup>11</sup>

$$z_\ell = \beta R b / \Delta. \quad (21)$$

We can use these five equations to define a symmetric stationary equilibrium. They determine the endogenous variables  $(\gamma, q, z_\ell, z_m, b)$ . Note that all other endogenous variables can be derived from these equilibrium values.

**Definition 1** *A symmetric stationary equilibrium is a list  $(\gamma, q, z_\ell, z_m, b)$  satisfying (17)-(21) with  $z_\ell \geq 0$  and  $z_m \geq 0$ .*

Let

$$\tilde{\Delta} = \frac{1 - \beta n + \pi / (1 + i_d)}{1/R - n\beta}. \quad (22)$$

Then we have the following

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<sup>11</sup>If the borrowing constraint is non-binding ( $\lambda_\ell = 0$ ), equation (15) reduces to  $V_b = \beta R$  implying from (5) that  $b = 0$  since we have  $\beta R < 1$ . Consequently, in any equilibrium where agents hold collateral it must be the case that the constraint is binding ( $\lambda_\ell > 0$ ) and so  $\ell = \bar{\ell} = Rb / [\phi_{+1} (1 + i)]$  implying  $\frac{\partial \ell}{\partial b} = R / [\phi_{+1} (1 + i)]$ .

**Proposition 1** *For any  $(i_\ell, i_d)$  with  $i_\ell \geq i_d \geq 0$  there exists a unique symmetric stationary equilibrium such that*

$$\begin{aligned} z_\ell > 0 \text{ and } z_m = 0 & \text{ if and only if } \Delta = 1 \\ z_\ell > 0 \text{ and } z_m > 0 & \text{ if and only if } 1 < \Delta < \tilde{\Delta} \\ z_\ell = 0 \text{ and } z_m > 0 & \text{ if and only if } \Delta \geq \tilde{\Delta}. \end{aligned}$$

Several points are worth mentioning. First, the critical element to verify in the proof is under which condition agents acquire collateral. They are willing to borrow at the standing facility if the borrowing rate is not too high, i.e., if  $\Delta < \tilde{\Delta}$ . Second, the critical value  $\tilde{\Delta}$  is increasing in  $R$  and  $\pi$ , and so is  $b$ . Agents increase their collateral holdings and hence finance a larger share of their consumption by borrowing if  $R$  or  $\pi$  are increased. Third, if  $\Delta = 1$  agents are not willing to hold money across periods. They just use collateral to borrow money to finance their consumption. This however does not mean that money is not used since it still plays the role of a medium of exchange in market 3. It only means that agents do not want to hold it across periods.

Given a real allocation  $(q(\Delta), b(\Delta))$  any pair  $(i_\ell, i_d)$  satisfying  $\Delta = \frac{1+i_\ell}{1+i_d}$  is consistent with this allocation. Thus, there are many ways to implement a given policy  $\Delta$ . The allocations only differ in the rate of inflation. This can be seen from (19) which can be written as follows

$$\frac{\gamma - \pi}{1 + i_d} = 1 - (1 - n)(\Delta - 1) \frac{z_\ell}{z_m}$$

Since the right-hand side is a constant for a given  $\Delta$  the inflation rate  $\gamma - 1$  is increasing in  $i_d$ .

In the introduction we have seen that the ECB (see Figure 1) reacts to changing economic condition by shifting the interest rate corridor  $\delta = i_\ell - i_d$ . An upwards shift of  $\delta$  increases  $\Delta$  and so reduces aggregate output  $q$  and borrowing  $z_\ell$ . Another way to tighten monetary policy is by increasing  $\delta$  since this also reduces  $q$  and  $b$ .

### 3.5 Optimal policy

We now derive the optimal policy. The central bank's objective is to maximize the expected lifetime utility of the representative agent. It does so by choosing lump sum transfers  $\pi$ , consumption  $q$  and collateral holding  $b$  to maximize (2) subject to constraint that its choice is consistent with the allocation given by (17)-(20). Given  $\pi$ , the policy is implemented by choosing  $\Delta$ .

Assume first that it is optimal to set  $\Delta \geq \tilde{\Delta}$ . In this case no agent is borrowing at the standing facility which implies that  $b = 0$ . Moreover, from (18) and (19)  $q$  satisfies

$$\tilde{q}(\pi) = u'^{-1} \left( \frac{1 - \beta n + \pi/(1 + i_d)}{\beta(1 - n)} \right).$$

Note that  $\tilde{q}$  is decreasing in  $\pi$  and that any  $\Delta \geq \tilde{\Delta}$  implements the same real allocation  $(b, q) = (0, \tilde{q})$ .

Now consider the largest  $q$  that the central bank can implement. From (17) the largest  $q$  is attained when  $\Delta = 1$ . It satisfies

$$\hat{q} = u'^{-1} \left[ \frac{1/(\beta R) - n}{1 - n} \right].$$

Thus, the policy  $\Delta = 1$  attains the allocation  $(b, q) = (\hat{q}/(\beta R), \hat{q})$  since no agent is holding money across period when  $\Delta = 1$ . Accordingly, the central bank is constrained to choose quantities  $q$  such that  $\hat{q} \geq q \geq \tilde{q}(\pi)$ .

Finally, it can be shown (see the proof of Proposition 1) that when  $1 \leq \Delta < \tilde{\Delta}$ ,  $b$  and  $q$  solve

$$\frac{1 - \beta R}{\beta R} = (1 - n) [u'(q)/\Delta - 1] \tag{23}$$

$$q = \beta R b F(\Delta; \pi) \tag{24}$$

where

$$F(\Delta; \pi) = \frac{1}{\Delta} \left[ 1 + \frac{(1 - n)(\Delta - 1)}{1 + \beta n(\Delta - 1) - \Delta/R + \pi/(1 + i_d)} \right].$$

Thus, the central bank is constrained to choose an allocation that satisfies (23) and (24) and so the central bank's maximization problem is

$$\begin{aligned} \max_{q,b,\pi} \quad & (1-n)[u(q)-q] + (\beta R - 1)b \\ \text{s.t.} \quad & q = \beta R b F\left(\frac{\beta R(1-n)u'(q)}{1-n\beta R}; \pi\right) \\ & \text{and } \hat{q} \geq q \geq \tilde{q}(\pi). \end{aligned} \tag{25}$$

where to derive (25) we use (23) to replace  $\Delta$  in (24).

**Proposition 2**  *$\pi = 0$  is optimal. Also, there exists a critical value  $\bar{R}$  such that if  $R < \bar{R}$ , then the optimal policy is  $\Delta \geq \tilde{\Delta}$ . Otherwise the optimal policy is  $\Delta \in (1, \tilde{\Delta})$ .*

The striking result of Proposition 2 is that it is never optimal to set a zero interest rate band  $\delta = i_\ell - i_d$  since the optimal interest rate band satisfies  $\Delta > 1$ . The reason is that for society the use of collateral is costly since  $\beta R - 1$  is negative. The benefit is that it increases consumption above  $q = \tilde{q}$ . The central bank thus faces a trade-off. It can encourage the use of costly collateral to increase consumption. The optimal policy simply equates the marginal benefit of additional consumption to the marginal cost of holding collateral. It is interesting to note that in contrast to collateral the use of fiat money is not costly for society since money can be produced without cost.

If  $R$  is small ( $R < \bar{R}$ ) it is optimal for the central bank to discourage the use of collateral.<sup>12</sup> It does so by implementing an interest rate policy that satisfies  $\Delta \geq \tilde{\Delta}$ . In contrast if the rate of return is high it sets  $\Delta \in (1, \tilde{\Delta})$  so that agents finance some of their consumption through borrowing at the standing facility. An increase in  $R$  reduces the optimal  $\Delta$ . In the limit as  $R \rightarrow 1/\beta$  the holding of collateral becomes costless and we now consider the optimal policy in this limiting case.

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<sup>12</sup>This is similar as in Lagos and Rocheteau (2004) albeit in a very different context. They construct a model where capital competes with fiat money as a medium of exchange. They show that when the socially efficient stock of capital is low (which is the case when the rate of return is low) a monetary equilibrium exists that dominates the nonmonetary one in terms of welfare. So in this case it would be optimal to discourage the use of capital as a medium of exchange.

These results are intuitive. The optimal monetary policy trades off the cost of holding collateral and the consumption flow from borrowing at the facility. When collateral is costly to hold, the central bank wants to discourage its use. This is achieved by increasing the cost of transforming collateral into money, that is by increasing the interest rate corridor. By modifying the liquidity properties of collateral, monetary policy affects the portfolio decision of agents and as a consequence the real allocation.

**Costless collateral** Holding collateral is costless when  $R = 1/\beta$  since the cost of acquiring one unit is equal to the discounted return  $\beta R$ . To avoid indeterminacies of the equilibrium allocation we consider the limiting allocation when the rate of return of the collateral satisfies  $R \rightarrow 1/\beta$ .<sup>13</sup> In this limiting case the critical value is  $\tilde{\Delta} = \frac{1-\beta n}{\beta-\beta n} > 1$  and Proposition 1 continues to hold. We define the allocation that is attained under the optimal policy as the limiting allocation that is attained when  $i_\ell \rightarrow i_d$ . We find the following results.

**Proposition 3** *With costless collateral, the optimal policy  $i_\ell \rightarrow i_d$  implements the first-best allocation  $q^*$ . The price level approaches infinity.*

The proof of the first part is an immediate consequence of equation (17) which implies that  $\lim_{\beta R \rightarrow 1} u'(q) = \Delta$ . Since the first-best allocation requires that  $u'(q) = 1$  the result is established.

To understand why the price level approaches infinity under the optimal policy note that if  $i_\ell = i_d > 0$ , then money is strictly dominated in return by collateral. The reason is that the collateral can costlessly be transformed into money and so any consumption level that can be achieved with money can be achieved with collateral

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<sup>13</sup>We consider the limiting allocation since at  $R = 1/\beta$  agents are indifferent of how much collateral they acquire even if they plan not to use it to obtain goods. If  $\lambda_\ell > 0$  agents are strictly better off by increasing their collateral holdings up to the amount where  $\lambda_\ell = 0$ . However, they are indifferent between any amount of collateral that yields  $\lambda_\ell = 0$ . In the limiting allocation attained when  $R \rightarrow 1/\beta$  agents acquire the smallest amount consistent with  $\lambda_\ell = 0$ .

at no additional cost. However, the collateral has the intrinsic return  $\beta R = 1$  while the return on money is  $\frac{\beta}{\gamma} < 1$ .<sup>14</sup> Consequently, the demand for money approaches zero. To encourage agents to hold the stock of money its price must approach zero. This immediately implies that  $p \rightarrow +\infty$  and therefore  $z_m = M_{+1}/p \rightarrow 0$ . Only at the Friedman rule  $i_\ell = i_d = 0$  the returns are equal and so agents are indifferent between holding money, collateral or both.

## 4 Trade in the money market

We now assume that  $\varepsilon > 0$ . Recall that at the beginning of the money market, agents receive a signal about the probability that they will become a consumer or a producer in the third market. With probability  $\sigma^k$  an agent receives the information that he will be a seller with probability  $n^k$ ,  $k = H, L$ .

We focus on the case where  $\varepsilon = n^H - n^L$  is small. This case captures the situation where agents' liquidity needs in the money market are not too different from each other and not too different from their initial beliefs. Consequently, they are reluctant to pledge all their collateral or to sell all their money holdings in the money market. Hence borrowing and lending constraints in the money market are not binding when  $\varepsilon$  is small (as shown below). As we will see later in this case the money market rate and the standing facility in our model closely matches the stylized fact mentioned in the introduction.

In what follows we look at a representative period  $t$ . Also, we assume the central bank does not make lump-sum transfers ( $\pi = 0$ ) since we have shown that this is optimal.

**Settlement** We let  $W(m, b, \ell, d, y)$  denote the expected value of entering the settlement stage with  $m$  units of money,  $b$  collateral,  $\ell$  loans,  $d$  deposits and private credit  $y$  where  $y > 0$  means that the agent has borrowed money in the money market of the

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<sup>14</sup>This follows from (18) together with  $u'(q) = \Delta$ .

previous period.  $Z(m, b)$  denotes the expected value from entering the money market with  $m$  units of money and  $b$  collateral.

In the first market, the problem of a representative agent is:

$$\begin{aligned} W(m, b, \ell, d, y) &= \max_{h, m_2, b_2} -h + Z(m_2, b_2) \\ \text{s.t. } \quad \phi m_2 + b_2 &= h + \phi m + Rb + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell - \phi(1 + i_m)y. \end{aligned}$$

where  $h$  is hours worked in market 1. The first-order conditions are

$$Z_m = \phi \tag{26}$$

$$Z_b \leq 1 \quad (= \text{ if } b > 0) \tag{27}$$

$Z_m \equiv \frac{\partial Z(m_2, b_2)}{\partial m_2}$  is the marginal value of taking an additional unit of money and  $Z_b \equiv \frac{\partial Z(m_2, b_2)}{\partial b_2}$  is the marginal value of taking additional collateral into the money market in period  $t$ . The envelope conditions are (6) and

$$W_y = -\phi(1 + i_m) \tag{28}$$

where  $W_y$  is the partial derivative of  $W(m, b, \ell, d, y)$  with respect to  $y$ .

**Money market** Let  $y^k$  be the amount of money acquired in the money market. An agent who has  $m$  money and  $b$  collateral at the opening of market 2 has expected lifetime utility

$$Z(m, b) = \sum_{k=H, L} \sigma^k V^k(m + y^k, b, y^k)$$

where  $y^k$  solves

$$\max_{y^k} V^k(m + y^k, b, y^k) \text{ s.t. } y^k \leq Rb / [\phi_{+1}(1 + i_m)] \text{ and } m + y^k \geq 0.$$

The first-order conditions are

$$V_m^k + V_y^k - \phi_{+1}\beta\lambda_{m\ell}^k + \phi_{+1}\beta\lambda_{md}^k = 0 \tag{29}$$

where  $\phi_{+1}\beta\lambda_{m\ell}^k$  is the multiplier on the borrowing constraint in the money market and  $\phi_{+1}\beta\lambda_{md}^k$  is the one on the lending constraint. Note that since in any equilibrium those agents who are likely to become sellers do not borrow money and those who are likely to become buyers do not lend money we have  $\lambda_{m\ell}^H = 0$  and  $\lambda_{md}^L = 0$  so that from (29)

$$V_m^H + V_y^H + \phi_{+1}\beta\lambda_{md}^H = 0 \quad (30)$$

$$V_m^L + V_y^L - \phi_{+1}\beta\lambda_{m\ell}^L = 0 \quad (31)$$

The marginal value of collateral is  $Z_b(m, b) = \sum_{k=H,L} \sigma^k [V_b^k + \sigma^k \beta R \lambda_{m\ell}^k / (1 + i_m)]$ . Then (31) gives us

$$Z_b(m, b) = \sum_{k=H,L} \sigma^k V_b^k + \sigma^L \beta R \frac{(V_m^L + V_y^L)}{\beta \phi_{+1} (1 + i_m)} \quad (32)$$

since in any equilibrium  $\lambda_{md}^L = \lambda_{m\ell}^H = 0$ .

The marginal value of money is  $Z_m(m, b) = \sum_{k=H,L} \sigma^k (V_m^k + \sigma^k \beta \phi_{+1} \lambda_{md}^k)$ . Then from (30) we have

$$Z_m(m, b) = \sigma^L V_m^L - \sigma^H V_y^H. \quad (33)$$

Thus, the marginal value of money at the beginning of the money market is equal to the expected value of using the money to buy goods in market 3,  $\sigma^L V_m^L$ , plus the expected value of lending it in the money market,  $-\sigma^H V_y^H$ .

Finally, the market clearing condition is

$$\sum_{k=H,L} \sigma^k y^k = 0. \quad (34)$$

**Goods market** At the beginning of market 3, an agent's state is revealed. Consider an agent of type  $k$  who received the signal that he will be a buyer with probability  $1 - n^k$  and a producer with probability  $n^k$ . Let  $q^k$  and  $q_s^k$  respectively denote the quantities consumed as a consumer and produced as a producer in market 3. Let  $\ell_b^k$  ( $\ell_s^k$ ) and  $d_b^k$  ( $d_s^k$ ) respectively denote the loan obtained from the central bank and the



amount of money deposited at the central bank by this agent in this market. If this agent holds  $m$  money,  $b$  collateral and private debt  $y$  at the opening of this market he has expected lifetime utility

$$\begin{aligned} V^k(m, b, y) = & (1 - n^k)[u(q^k) + \beta W(m - pq^k + \ell^k, b, \ell^k, 0, y)] \\ & + n^k[-q_s^k + \beta W(m + pq_s^k - d^k, b, 0, d^k, y)] \end{aligned}$$

where  $q^k, q_s^k, \ell_s^k, \ell_b^k, d_s^k$  and  $d_b^k$  are chosen optimally as described in Section 3. The only difference is that the constraints in the goods market now take into account an agent's borrowing or lending  $y^k$  in the money market as follows

$$\ell^k \leq \bar{\ell}^k \equiv Rb / [\phi_{+1} (1 + i_\ell)] - y^k / \hat{\Delta} \quad (35)$$

$$pq^k \leq m + \ell^k \quad (36)$$

$$d^k \leq m \quad (37)$$

where  $\hat{\Delta} = \frac{1+i_\ell}{1+i_m}$ . Note that in equilibrium  $m = M + y^k$ . The quantity  $\bar{\ell}^k$  is still the maximal amount that a buyer can borrow from the central bank. If the agent has borrowed money ( $y^k > 0$ ), the maximal loan size is reduced by  $y^k (1 + i_m) / (1 + i_\ell)$ . In contrast, if the agent has lent money ( $y^k < 0$ ), it is increased accordingly.

Finally, using the fact that  $\lambda_\ell^k = u'(q^k) (1 + i_d) - (1 + i_\ell)$ ,  $\lambda_q^k = \lambda_\ell^k + i_\ell$  and  $\lambda_d^k = i_d$  the marginal value of money, the marginal value of collateral and the marginal value of private debt in market 3 can be written as follows

$$V_m^k = \beta \phi_{+1} (1 + i_d) \{1 + (1 - n^k) [u'(q^k) - 1]\} \quad (38)$$

$$V_b^k = \beta R \{1 + (1 - n^k) [u'(q^k) / \Delta - 1]\} \quad (39)$$

$$V_y^k = -\beta \phi_{+1} (1 + i_m) \{1 + (1 - n^k) [u'(q^k) / \Delta - 1]\}. \quad (40)$$

**Endogenous money supply** Finally, we need to adjust equation (1) to take into account how the money market affects the evolution of the stock of money across periods. The new equation is

$$M_{+1} = M - [\sigma^H (1 - n^H) \ell^H + \sigma^L (1 - n^L) \ell^L] i_\ell + [\sigma^H n^H d^H + \sigma^L n^L d^L] i_d \quad (41)$$

where  $\ell^k = Rb / [\phi_{+1} (1 + i_\ell)] - y^k / \hat{\Delta}$  and  $d^k = M + y^k + pq_s^k$ . Using the market clearing conditions in the good and money market we can write this equation as follows

$$M_{+1}/M = 1 + i_d - (i_\ell - i_d) [\sigma^L(1 - n^L)\ell^L/M + \sigma^H(1 - n^H)\ell^H/M]. \quad (42)$$

It is interesting to compare (42) with (19) (when  $\pi = 0$ ). As before the entire stock of money earns interest  $i_d$ . The only difference is the amount of loans that the central bank provides. Without money market the amount is  $(1 - n)\ell/M$  with a money market it is  $[\sigma^L(1 - n^L)\ell^L/M + \sigma^H(1 - n^H)\ell^H/M]$ .

#### 4.1 Symmetric stationary equilibrium

We again focus on stationary equilibria which requires that  $M_{+1}/M = \phi/\phi_{+1} = \gamma$ . Use equations (9), (32), (38)-(40) and rearrange to write the first-order condition (5) as follows

$$\frac{1-\beta R}{\beta R} = \sigma^H (1 - n^H) [u'(q^H)/\Delta - 1] + \sigma^L \frac{\hat{\Delta}}{\Delta} \left\{ \frac{\hat{\Delta}-\Delta}{\Delta} + (1 - n^L) [u'(q^L) - 1] \right\}. \quad (43)$$

Use (9), (33), (38)-(40) and rearrange to write the first-order condition (4) as follows

$$\frac{\gamma - \beta(1+i_d)}{\beta(1+i_d)} = \sigma^L(1 - n^L) [u'(q^L) - 1] + \sigma^H \frac{\Delta}{\hat{\Delta}} \left\{ \frac{\Delta-\hat{\Delta}}{\Delta} + (1 - n^H) [u'(q^H)/\Delta - 1] \right\}. \quad (44)$$

Then combine (43) with (44) to get the Fisher equation

$$R\gamma = 1 + i_m. \quad (45)$$

By defining  $r \equiv R - 1$  and  $\pi \equiv \gamma - 1$  we get the standard expression for the Fisher equation  $(1 + r)(1 + \pi) = 1 + i_m$ . It is interesting to note that the nominal interest rate of the Fisher equation is the money market rate  $i_m$  and not the interest rates from the standing facilities.

In any equilibrium the budget constraints hold with equality and so

$$pq^k = m^k + \ell^k = M + y^k + \ell^k, \quad k = H, L. \quad (46)$$

Use (46) to substitute  $y^H$  and  $y^L$  in the money market's market clearing condition (34) and rearrange to get

$$\sigma^H q^H + \sigma^L q^L = z_m + \frac{\beta R b}{\Delta}. \quad (47)$$

We can combine (34) with (46) to get the real amount of balances acquired ( $z^L$ ) or sold ( $z^H$ ) on the money market

$$z^H = -\sigma^L (q^L - q^H) \left( \frac{\hat{\Delta}}{\hat{\Delta} - 1} \right) = -\frac{\sigma^L}{\sigma^H} z^L. \quad (48)$$

Finally, we need to get an expression for the real stock of money. To derive  $z_m$  use the Fisher equation (45) to write (42) as follows

$$\frac{R\hat{\Delta} - \Delta}{R\hat{\Delta}(\Delta - 1)} = \sigma^L(1 - n^L)\ell^L/M + \sigma^H(1 - n^H)\ell^H/M].$$

Then, use (46) to substitute  $\ell^H$  and  $\ell^L$  and rearrange to get

$$\frac{R\hat{\Delta} - \Delta}{R\hat{\Delta}(\Delta - 1)} = -(1 - n) + \frac{1}{z_m} [\sigma^L(1 - n^L)q^L + \sigma^H(1 - n^H)q^H - \sigma^L(n^H - n^L)z^L].$$

Finally use (48) and solve for  $z_m$  to get

$$z_m = \left( \frac{\hat{\Delta}}{\hat{\Delta} - 1} \right) \frac{\{(\hat{\Delta} - 1)[\sigma^L(1 - n^L)q^L + \sigma^H(1 - n^H)q^H] - (n^H - n^L)\sigma^L\sigma^H(q^L - q^H)\hat{\Delta}\}R(\Delta - 1)}{R\hat{\Delta} - \Delta + (1 - n)R\hat{\Delta}(\Delta - 1)}. \quad (49)$$

Note that equations (43) - (49) must hold in any equilibrium. We now consider the case where no short-selling constraint is binding in the money market.

When no short-selling constraint is binding in the money market  $\lambda_{md}^H = \lambda_{m\ell}^L = 0$  and so from (29)  $V_m^L + V_y^L = V_m^H + V_y^H = 0$ . Then, (38) and (40) imply

$$u'(q^k) = \frac{n^k}{1 - n^k} \frac{(\Delta - \hat{\Delta})}{(\hat{\Delta} - 1)}, \quad k = H, L.$$

Using these expressions to substitute  $u'(q^H)$  and  $u'(q^L)$  in (43) and solving for  $\hat{\Delta}$  yields

$$\hat{\Delta} = \frac{\Delta}{n\beta R(1 - \Delta) + \Delta}. \quad (50)$$

Substitute  $\hat{\Delta}$  back in the above equations to get the quantities

$$u'(q^k) = \frac{n^k}{1 - n^k} \Delta \frac{1 - n\beta R}{n\beta R}, \quad k = H, L. \quad (51)$$

**Definition 2** *A symmetric stationary equilibrium where no short-selling constraint is binding in the money market is a time-invariant list  $(\hat{\Delta}, q^L, q^H, z^L, z^H, z_m, b, \gamma)$  satisfying (45) - (51) with  $b \geq 0$ ,  $z^L < \beta R b \hat{\Delta} / \Delta$  and  $z^H > -z_m$ .*

Then we have the following:

**Proposition 4** *For any  $1 < \Delta < \tilde{\Delta}$  there exists a critical value  $\varepsilon_1 > 0$  defined in the proof such that if  $\varepsilon < \varepsilon_1$  a symmetric monetary equilibrium exists where no short-selling constraint in the money market binds.*

Note first that the system of equations (45) - (51) can be solved recursively. Equations (50) and (51) yield  $\hat{\Delta}$ ,  $q^L$  and  $q^H$ . Using these values we can then derive  $(z^L, z^H, z_m, b, \gamma)$  from the remaining equations. One then has to check that the required inequalities hold. The inequality  $b \geq 0$  simply requires that policy is such that agents have an incentive to acquire collateral which is satisfied whenever  $\Delta < \tilde{\Delta}$  (defined by (22)). The inequality  $z^L < \beta R b \hat{\Delta} / \Delta$  requires that those agents who are likely to become buyers are not pledging all their collateral to acquire money in the money market and the inequality  $z^H > -z_m$  requires that those agents who are likely to become sellers are not selling all their money.

## 4.2 Policy implications

The following policy implications emerge from the model. First, the money market rate  $i_m$  changes proportionally to a shift of the interest rate corridor. To see this we can write (50) as follows

$$i_m = i_\ell - n\beta R\delta$$

where  $\delta = i_\ell - i_d$  is the spread of the interest rate corridor. Obviously,  $i_m$  increases in  $i_\ell$  one for one when  $\delta$  is kept constant. Thus, a shift of the corridor by 50 basis points will increase the money market rate by 50 basis points.

Second, assume that  $n\beta R = 1/2$ . For example assume  $n = 1/2$  and  $\beta R \rightarrow 1$ . The first assumption means that one half of the agents are borrowing and the other half are providing cash in the money market. The second assumption means that holding collateral has no cost. Then, when  $n\beta R = 1/2$  the money market rate is  $i_m = \frac{i_\ell + i_d}{2}$ . That is the money market rate lies exactly on the target (or policy) interest rate. Therefore, the first and the second features of our model exactly match the behavior of the overnight money market rate of the channel system operated by the Reserve Bank of New Zealand. This can be seen from Figure 2 in the introduction. Our model suggests that the reason for this is that the short-selling constraints of the private banks in New Zealand are most of the time not binding and that holding collateral is not very costly, i.e.,  $\beta R \rightarrow 1$ .

Third, as mentioned in the introduction, the Euro money market rate tends to be above the target rate. Our model has a simple explanation for this observation. With  $n = 1/2$  and  $\beta R < 1$  we have  $i_m = \frac{i_\ell(2-\beta R) + i_d\beta R}{2} > \frac{i_\ell + i_d}{2}$ . Thus, when it is costly to hold collateral the money market rate tends to be above the target rate.

Fourth, aggregate consumption is decreasing in  $\Delta$  since  $q^H$  and  $q^L$  are decreasing in  $\Delta$ . It is also decreasing when the central bank shifts the interest rate corridor upwards because such a shift increases  $\Delta$ . Thus the model is consistent with the notion that the central bank is tightening its policy when it shifts the corridor upwards (holding everything else constant).

Fifth, from the Fisher equation  $\gamma = \frac{1+i_m}{R}$ , the growth rate of the money supply is increasing in  $i_\ell$  since  $i_m$  is increasing in  $i_\ell$ . In contrast, an increase in  $i_d$ , holding everything else constant, decreases the rate of inflation.

Finally, a central bank can tighten its policy without changing its target rate by simply increasing the corridor symmetrically around the target rate. This makes it more costly for banks to access the standing facilities and therefore it is tightening a

central bank's policy regime.

## 5 Conclusion

We have analyzed the theoretical properties of a channel system of interest rate control in a dynamic general equilibrium model with infinitely-lived agents and a central bank. With this model we could match the stylized facts regarding the use of channel systems by central banks. First, all central banks set a strictly positive corridor. Second, central banks typically react to changing economic conditions by shifting the interest-rate corridor. Third, the money market rate tends to be in the middle or above the middle of the corridor. We have also shed light on the role of collateral and the link between the corridor and the conditions prevailing in the money market.

While our paper is a first step towards analyzing monetary policy implementation in a channel system, many aspects have remained unexplored. In particular, optimal monetary policy in a channel system under stress and aggregate shocks are left for future research.

## 6 APPENDIX

### 6.1 Background<sup>15</sup>

To understand some of the features of our environment it is useful to have some information on how the money market functions and on monetary policy procedures at central banks that operate a standing facility. This section does not aim at being general and we will therefore concentrate on the case of the euro money markets and the ECB's operating procedures.

**Operating procedures of the ECB** The ECB has two main instruments for the implementation of its monetary policy. First, it conducts weekly main refinancing operations that are collateralized loans with a one week maturity. Main refinancing operations are implemented using a liquidity auction where banks bid for liquidity. Bids consist of an amount of liquidity and an interest rate. The total amount to be allocated is announced before the auction. Following the auction, the ECB allocates liquidity according to the bided rates, in a descending order. The minimum bid rate is the main policy rate used by the ECB to implement monetary policy.

Second, the ECB offers a standing facility with a lending rate that is 100 basis points higher than its minimum bid rate and a deposit rate, which is 100 basis points below its minimum bid rate. At the lending facility, liquidity is provided either in the form of overnight repurchase agreements or as overnight collateralized loans whereby the ownership of the asset is retained by the debtor. In both case, banks have to resort to safe eligible assets as defined by the ECB. Eligible banks can access the standing facilities at any time of the day. The use of the standing facility largely depends on banks' activities on the euro money markets during the day.

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<sup>15</sup>This section draws heavily on materials from ECB (2005), ECB (2004), BIS (2003) and Hartmann, Manna and Manzanares (2001).

**The euro money markets** There are two segments for the euro money market. The first segment is the unsecured money market, where banks borrow and lend cash to each other without resorting to collateral. The reference interest rate on the unsecured money market is the Euro Overnight Index Average (EONIA) calculated by the ECB. The second segment is the secured money market where agents lend at different maturities against collateral. This is the largest money market segment. There are several reference interest rates depending on maturities (Euro interbank offered rates, or Euribors) and whether the collateral pledged belong to a general collateral pool (Euripo).

Transactions on both segments of the money market are settled using the two large-value payment systems operating in the euro area, the Trans-European Automated Real-Time Gross settlement Express Transfer system (TARGET) and Euro1. These large value payment systems are essential in finalizing the transfer of funds for transactions taking place in money markets. Therefore, the opening and closing hours of money markets are closely related to the operating hours of these payment systems.

TARGET settles payments with immediate finality in central bank money and operates between 7am and 6pm C.E.T. with a cut-off time of 5pm for customer payments.<sup>16</sup> Eligible institutions hold accounts at TARGET, which are debited or credited depending on market participants' orders. Intraday credit is provided free of charge as long as it is fully collateralized. Banks may also access the deposit or lending facilities after making a request at the latest 30 minutes after the actual closing time of TARGET.<sup>17</sup> After the close of TARGET, an overdraft position on a bank's TARGET account is automatically transformed into an overnight loan via a recourse to the lending facility, again against eligible assets.

Euro1 is a private large-value payment systems offered by the Euro Banking As-

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<sup>16</sup>The unsecured segment opens around 8am in the morning and closes around 5:45pm.

<sup>17</sup>On the last Eurosystem business day of a minimum reserve maintenance period, the deposit facility can be accessed for 60 minutes after the actual closing of TARGET.



sociation (EBA). Euro1 functions as a sort of netting system, whereby on each settlement day, at any given time, each participant will have only one single payment obligation or claim with respect to the community of other participant as joint creditors/debtors. In particular, there is no bilateral payments, claims or obligations between participants. Euro1 settles in central bank money at the ECB at the end of the day. After the cut-off time of 4pm C.E.T., clearing banks with debit positions will pay their single obligations into the EBA settlement account at the ECB through TARGET. After all amounts due have been received, the ECB will pay the clearing banks with credit positions also through TARGET.

In this paper, we will model two specific features of the description above. First, banks cannot carry overnight overdrafts on their TARGET accounts, and they have to borrow either on the money markets or at the lending facility in order to cover their TARGET positions. When TARGET closes, euro money markets are also closed. As a consequence, the central bank standing facility is, at the end of the day, the only recourse to overnight liquidity. Also, since participants can access the standing facility 30 minutes after the close of target, any late payments received on a TARGET account can be deposited at the standing facility of the ECB. In the first part of the paper we model this aspect of the liquidity management problem. Second, banks can predict when a payment is due or incoming so that with a well functioning money market, the likelihood to resort to the standing facilities should be small. However, there may be unexpected payments to be made that can force banks to hold an overdraft on their TARGET account. In the second part of the paper, we adjunct a money market to the model. There, banks will be able to trade their liquidity when they are confident that they will end up the day with a credit on their central bank account. Given it is the most important segment of the money market, we concentrate on the secured interbank money market. At this stage we will abstract from modelling liquidity injections in an interbank market.

## 6.2 Welfare

In this Appendix we show that if the central bank's objective is to maximize the expected discounted utility of the representative agent, the central bank's objective is to maximize (2). To derive (2) we must first calculate hours worked in market 1. The money holdings at the opening of the first market are  $\tilde{m} = 0$  having bought and  $\tilde{m} = m + pq_s$  having sold. Hence, hours worked are

$$\begin{aligned} h_b &= \phi[m_{+1} + (1 + i_\ell)\ell] - (R - 1)b - \pi M \\ h_s &= \phi[m_{+1} - (1 + i_d)(m + pq_s)] - (R - 1)b - \pi M \end{aligned}$$

Since  $h = nh_s + (1 - n)h_b$ , by using (1) and rearranging we get

$$\begin{aligned} h &= -(R - 1)b + (1 - n)\phi\ell - n\phi(m + pq_s) + \phi m \\ &= -(R - 1)b + (1 - n)\phi\ell + (1 - n)\phi m - n\phi pq_s \\ &= -(R - 1)b. \end{aligned}$$

since  $pq = m + \ell$  and  $q_s = \frac{1-n}{n}q$ . Then, welfare is given by

$$\begin{aligned} \mathcal{W} &= -b + (1 - n)[u(q) - q] + \sum_{j=1}^{\infty} \beta^j \{(1 - n)[u(q) - q] + (R - 1)b\} \\ &= \frac{(1 - n)[u(q) - q] + (\beta R - 1)b}{1 - \beta} \end{aligned}$$

## 6.3 Proofs

**Proof of Proposition 1.** For ease of exposition we assume  $\pi = 0$ . The proof can be easily replicated when  $\pi > 0$ . We first prove the *only if* part. Assume first  $z_\ell = 0$  and  $z_m > 0$ . Then from (18) and (19) we get

$$\frac{1 - \beta}{\beta} = (1 - n)[u'(q) - 1]. \quad (52)$$

and from (17) we have

$$\frac{1 - R\beta}{R\beta} \geq (1 - n)[u'(q)/\Delta - 1]. \quad (53)$$

Use (53) to replace  $u'(q)$  in (52) and rearrange to get  $\Delta \geq \tilde{\Delta}$ .

Assume now that  $z_\ell > 0$  and  $z_m > 0$ . Then from (19)  $z_\ell > 0$  implies that  $1 + i_d > \gamma$ . Use (18) to replace  $\gamma$  and rearrange to get  $\Delta < \tilde{\Delta}$ . Next divide (19) by  $1 + i_d$  and solve for  $\Delta$  to get

$$\Delta = 1 + \frac{z_m}{z_\ell} \frac{1 + i_d - \gamma}{(1 - n)(1 + i_d)} > 1$$

since  $1 + i_d > \gamma$ . Hence we have  $1 < \Delta < \tilde{\Delta}$  if  $z_\ell > 0$  and  $z_m > 0$ .

Finally, assume that  $z_\ell > 0$  and  $z_m = 0$ . Then, the previous equation immediately implies that  $\Delta = 1$ .

We now prove the *if* part. From (18) and (19) we get

$$1 - n\beta - \beta(1 - n)u'(q) = (1 - n)(\Delta - 1)\frac{z_\ell}{z_m}. \quad (54)$$

and from (17) we have

$$\Delta \left( \frac{1}{R} - n\beta \right) \geq \beta(1 - n)u'(q) \quad (55)$$

Assume first that  $1 < \Delta < \tilde{\Delta}$ . Use (54) to rewrite (55) as follows

$$1 - n\beta - \Delta \left( \frac{1}{R} - n\beta \right) \leq (1 - n)(\Delta - 1)\frac{z_\ell}{z_m}.$$

Rearrange to get

$$0 < \tilde{\Delta} - \Delta \leq \frac{(1 - n)(\Delta - 1)}{(1/R - n\beta)} \frac{z_\ell}{z_m}.$$

Hence,  $1 < \Delta < \tilde{\Delta}$  implies  $\frac{z_\ell}{z_m} > 0$ .

Assume next that  $\Delta \geq \tilde{\Delta}$ . Then from (54) we have

$$1 - n\beta - \beta(1 - n)u'(q) \geq (1 - n)(\tilde{\Delta} - 1)\frac{z_\ell}{z_m}.$$

Then  $z_\ell > 0$  immediately implies that

$$0 > \tilde{\Delta} - \Delta \geq \frac{(1 - n)(\tilde{\Delta} - 1)}{(1/R - n\beta)} \frac{z_\ell}{z_m}.$$

a contradiction. Hence  $\Delta \geq \tilde{\Delta}$  implies  $z_\ell = 0$ .

**Existence and uniqueness when  $\tilde{\Delta} \leq \Delta$ :** In this case  $z_\ell = b = 0$  and from (19)  $\gamma = 1 + i_d$ . Then, from (18) and (19) we get (52). Since right-hand side of (52) is strictly decreasing in  $q$  there exists a unique  $q$  that solves (52). Finally, from (20) we have  $z_m = q$ .

**Existence and uniqueness when  $1 < \Delta < \tilde{\Delta}$ :** The system of equations (17)-(20) with  $z_\ell = \beta Rb/\Delta$  can be reduced as follows. Equations (20) and  $z_\ell = \beta Rb/\Delta$  imply  $z_m = q - \beta Rb/\Delta$ . Then, multiply both sides of (19) by  $z_m$  and replace  $z_m$  to get

$$(q - \beta Rb/\Delta) [\gamma - (1 + i_d)] = -(1 - n)z_\ell(i_\ell - i_d)$$

Use (18) to eliminate  $\gamma$  and rearrange to get

$$(q - \beta Rb/\Delta) \{1 - (1 - n)\beta[u'(q) - 1] - \beta\} = (1 - n)\frac{\beta Rb}{(1 + i_\ell)}(i_\ell - i_d)$$

Hence, an equilibrium is defined by the following two equations:

$$\begin{aligned} \frac{1}{R\beta} &= (1 - n)u'(q)/\Delta + n \\ (q - \beta Rb/\Delta) \{1 - (1 - n)\beta[u'(q) - 1] - \beta\} &= (1 - n)\frac{\beta Rb}{(1 + i_\ell)}(i_\ell - i_d) \end{aligned}$$

We can use the first equation to replace for  $u'(q)$  in the second to get

$$\begin{aligned} \frac{1}{R\beta} &= (1 - n)u'(q)/\Delta + n \\ q &= \beta RbF(\Delta) \end{aligned}$$

If we substitute  $q$  in the first expression, we get

$$\frac{1}{R\beta} = (1 - n)u'[\beta RbF(\Delta)]/\Delta + n \equiv RHS \quad (56)$$

The left-hand side of (56) is constant while the right-hand side is decreasing in  $b$  for a given  $1 \leq \Delta < \tilde{\Delta}$ . Moreover, we have  $\lim_{b \rightarrow 0} RHS = +\infty$  and  $\lim_{b \rightarrow \infty} RHS = n < \frac{1}{R\beta}$ . Hence, for any policy  $\Delta$  with  $1 \leq \Delta < \tilde{\Delta}$  a unique  $b > 0$  exists. Then, from (24)

a unique value for  $q$  exists. Accordingly a unique symmetric stationary equilibrium exist.

Finally, we have  $\lim_{\Delta \rightarrow \tilde{\Delta}} F(\Delta) = +\infty$  and so  $b \rightarrow 0$ . ■

**Proof of Proposition 2.** We first show that  $\pi = 0$  is optimal. Note that  $\hat{q}$  is independent of  $\pi$  and  $\tilde{q}$  is decreasing in  $\pi$ . Therefore, increasing  $\pi$  does not improve the set of attainable allocations. Second,  $F(\Delta; \pi)$  is decreasing in  $\pi$ , so that given  $q$ ,  $i_\ell$  and  $i_d$ , increasing  $\pi$  would increase  $b$  and therefore would reduce welfare. Hence, since  $\pi \geq 0$ , it is optimal to set  $\pi$  to zero.

We now assume  $\pi = 0$ . Substituting (25) into the objective function the problem becomes

$$\begin{aligned} \max_q \quad & (1-n)[u(q) - q] + (\beta R - 1) \frac{q}{\beta R F\left(\frac{R\beta(1-n)u'(q)}{1-nR\beta}\right)} \\ \text{s.t.} \quad & \hat{q} \geq q \geq \tilde{q}. \end{aligned}$$

After rearranging, the first-order condition is

$$(1-n)[u'(q) - 1] + \frac{1-\beta R}{\beta R F(\Delta)} \left[ \frac{F'(\Delta) \Delta u''(q) q}{F(\Delta) u'(q)} - 1 \right] = \hat{\lambda} - \tilde{\lambda}$$

where  $\hat{\lambda}$  is the multiplier of the first inequality and  $\tilde{\lambda}$  the one of the second inequality.

Consider the first-order condition and note that

$$\Delta(q) = \frac{R\beta(1-n)u'(q)}{1-nR\beta}.$$

Suppose that the optimal  $q$  is such that  $\Delta = 1$ , i.e.,  $q = \hat{q}$ . In this case  $\tilde{\lambda} = 0$  and  $\hat{\lambda} > 0$  implying that  $\Theta(\hat{q}, R) > 0$ . Then we have  $F(1) = 1$ ,  $F'(1) = \frac{1-nR}{R-1}$  and so

$$\Theta(\hat{q}, R) = \frac{1-\beta R}{\beta R} \frac{1-nR}{R-1} \frac{u''(\hat{q})\hat{q}}{u'(\hat{q})} < 0$$

which is a contradiction. Thus, in any equilibrium  $q < \hat{q}$  implying  $\Delta > 1$ .

Now suppose that the optimal  $q$  is such that  $\Delta = \tilde{\Delta}$ , i.e.,  $q = \tilde{q}$ . In this case  $\tilde{\lambda} > 0$  and  $\hat{\lambda} = 0$  implying that  $\Theta(\tilde{q}, R) < 0$ . One can show that  $\lim_{\Delta \rightarrow \tilde{\Delta}} F(\Delta) = \infty$ ,

$\lim_{\Delta \rightarrow \hat{\Delta}} F'(\Delta) = \infty$ ,  $\lim_{\Delta \rightarrow \hat{\Delta}} \frac{F'(\Delta)\Delta}{F(\Delta)} = \infty$  and  $\lim_{\Delta \rightarrow \hat{\Delta}} \frac{F'(\Delta)\Delta}{F(\Delta)F(\Delta)} = \frac{(1-1/R)}{(1/\Delta)^2(1-n)(\Delta-1)^2}$ . Moreover,  $(1-n)[u'(q) - 1] = 1/\beta - 1$ . Accordingly, we get

$$\Theta(\tilde{q}, R) = 1/\beta - 1 + \frac{1 - \beta R}{\beta R} \frac{R(1 - \beta n)^2}{(R - 1)(1 - n)} \frac{u''(\tilde{q})\tilde{q}}{u'(\tilde{q})}$$

Consider first  $R \rightarrow 1$ . Then we have  $\lim_{R \rightarrow 1} \Theta(\tilde{q}, R) = -\infty$ . Now consider  $R \rightarrow 1/\beta$ . Then we have  $\lim_{R \rightarrow 1/\beta} \Theta(\tilde{q}, R) = 1/\beta - 1 > 0$ . Since  $\frac{1 - \beta R}{\beta} \frac{(1 - \beta n)^2}{(R - 1)(1 - n)}$  is monotonically decreasing in  $R$  we have a unique critical value  $\bar{R}$  such that  $\Theta(\tilde{q}, \bar{R}) = 0$ . Thus if  $R < \bar{R}$ ,  $q = \tilde{q}$  and if  $R > \bar{R}$ ,  $q$  solves  $\Theta(q, R) = 0$ . ■

**Proof of Proposition 4.** The first thing to note is that the system of equations (45) - (51) can be solved recursively as described in the text. It remains to show under which conditions the short selling constraints in the money market are nonbinding. Thus, we need to verify that  $y^k < Rb/[\phi_{+1}(1 + i_m)]$  and that  $m + y^k > 0$ . Using the seller's first-order condition and dividing by  $p$  we can write these conditions as follows

$$z^k < \beta Rb\hat{\Delta}/\Delta \text{ and } z_m + z^k > 0$$

Since  $z^L > z^H$  it is sufficient to check that  $z^L < \beta Rb\hat{\Delta}/\Delta$ . Along the same line, since  $z^L > z^H$  it is sufficient to check that  $z^H > -z_m$ .

Let us first consider whether  $z^H > -z_m$ . From (48) and (49)  $z^H > -z_m$  if

$$\sigma^L(q^L - q^H) < \frac{(\hat{\Delta} - 1)[\sigma^L(1 - n^L)q^L + \sigma^H(1 - n^H)q^H] - \sigma^L\sigma^H(q^L - q^H)(n^H - n^L)\hat{\Delta}}{\Phi}$$

where  $\Phi = (R\hat{\Delta} - \Delta)/[R(\Delta - 1)] + (1 - n)\hat{\Delta}$ . Note that  $\Phi > (1 - n)\hat{\Delta}$  since  $R\hat{\Delta} > \Delta$ .

Then  $n^H - n^L = \varepsilon$  and  $\sigma^L n^L + \sigma^H n^H = n$  yield  $n^H = n + \sigma^L \varepsilon$  and  $n^L = n - \sigma^H \varepsilon$ . Using these relations and rearranging yields

$$q^L - q^H < \frac{(\hat{\Delta} - 1)(1 - n)\left(\frac{\sigma^H}{\sigma^L}q^H + q^L\right) - \varepsilon\sigma^H(q^L - q^H)}{\Phi}$$

Divide by  $q^H$  and rearrange to get

$$\frac{q^L}{q^H} \left[ \Phi - (\hat{\Delta} - 1)(1 - n) + \varepsilon\sigma^H \right] < \Phi + \frac{\sigma^H}{\sigma^L} (\hat{\Delta} - 1)(1 - n) + \varepsilon\sigma^H$$

The left-hand side is larger than zero since  $\Phi > (1 - n)\hat{\Delta}$ . Moreover, it is strictly smaller than the right-hand side at  $\varepsilon = 0$  (since  $q^L = q^H$  at  $\varepsilon = 0$ ). Then, divide the inequality by  $\left[\Phi - (1 - n)(\hat{\Delta} - 1) + \sigma^H \varepsilon\right]$  to get

$$\frac{q^L}{q^H} < \frac{\Phi + \frac{\sigma^H}{\sigma^L}(\hat{\Delta} - 1)(1 - n) + \sigma^H \varepsilon}{\Phi - (\hat{\Delta} - 1)(1 - n) + \sigma^H \varepsilon}$$

The left hand side is increasing in  $\varepsilon$  and the right-hand side is decreasing. Therefore there is a unique  $\tilde{\varepsilon}_1$  such that  $z_H > -z_m$  when  $\varepsilon < \tilde{\varepsilon}_1$ .

We next check  $\beta Rb\hat{\Delta}/\Delta > z^L$ . From  $\sigma^H q^H + \sigma^L q^L = z_m + \frac{\beta Rb}{\Delta}$  we need  $\sigma^H q^H + \sigma^L q^L > z_m + z^L/\hat{\Delta}$ , or replacing for  $z_m$  and  $z^L$ , and rearranging we need

$$\sigma^H q^H + \sigma^L q^L > \frac{\hat{\Delta}\{(\hat{\Delta}-1)[\sigma^L(1-n^L)q^L + \sigma^H(1-n^H)q^H] - (n^H - n^L)\sigma^L\sigma^H(q^L - q^H)\hat{\Delta}\}}{(\hat{\Delta}-1)\Phi} + \frac{\sigma^H(q^L - q^H)}{(\hat{\Delta}-1)}$$

Multiply through by  $(\hat{\Delta} - 1)$  and arrange to obtain

$$(\sigma^H q^H + \sigma^L q^L) \hat{\Delta} - q^L > \frac{\hat{\Delta}\{(\hat{\Delta}-1)[\sigma^L(1-n^L)q^L + \sigma^H(1-n^H)q^H] - (n^H - n^L)\sigma^L\sigma^H(q^L - q^H)\hat{\Delta}\}}{\Phi}$$

Use  $n^H = n + \sigma^L \varepsilon$  and  $n^L = n - \sigma^H \varepsilon$  to substitute  $n^H$  and  $n^L$  and rearrange to get

$$\left(\sigma^H q^H + \sigma^L q^L - \frac{q^L}{\hat{\Delta}}\right) \Phi > (1 - n)(\sigma^L q^L + \sigma^H q^H)(\hat{\Delta} - 1) - \sigma^L \sigma^H (q^L - q^H) \varepsilon$$

This expression is satisfied at  $\varepsilon = 0$  since we have  $\Phi > (1 - n)\hat{\Delta}$ . Dividing both sides by  $\sigma^H q^H + \sigma^L q^L$ , and rearranging gives

$$\frac{\Phi}{\hat{\Delta}(\sigma^H \frac{q^H}{q^L} + \sigma^L)} < \Phi - (\hat{\Delta} - 1)(1 - n) + \frac{\sigma^L \sigma^H \varepsilon \left(1 - \frac{q^H}{q^L}\right)}{\sigma^H \frac{q^H}{q^L} + \sigma^L}$$

Since  $\frac{q^H}{q^L}$  is decreasing in  $\varepsilon$ , the left-hand side is increasing in  $\varepsilon$  and the right-hand side is also increasing in  $\varepsilon$ . Therefore, given this constraint does not bind at  $\varepsilon = 0$ , either it never binds or it binds for some  $\varepsilon > \varepsilon_1$ . Thus, if  $\varepsilon < \varepsilon_1 = \min\{\tilde{\varepsilon}_1, \varepsilon_1\}$  a unique equilibrium exists where no short-selling constraint binds. ■

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